SUDOKU Colorings of the Hexagonal Bipyramid Fractal

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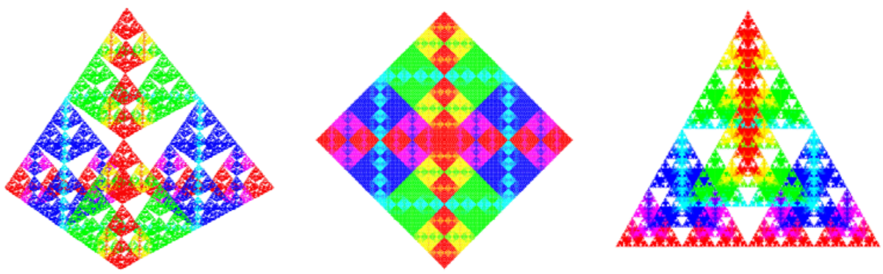
Abstract. The hexagonal bipyramid fractal is a fractal in three dimensional space, which has fractal dimension two and which has six square projections. We consider its 2nd level approximation model, which is composed of 81 hexagonal bipyramid pieces. When this object is looked at from each of the 12 directions with square appearances, the pieces form a 9×9 grid of squares which is just the grid of the SUDOKU puzzle. In this paper, we consider colorings of the 81 pieces with 9 colors so that it has a SUDOKU solution pattern in each of the 12 appearances, that is, each row, each column, and each of the nine 3×3 blocks contains all the 9 colors in each of the 12 appearances. We show that there are 140 solutions modulo change of colors, and, if we identify isomorphic ones, we have 30 solutions. We also show that SUDOKU coloring solutions exist for every level $2n$ approximation models ($n \geq 1$).

1 Introduction

The Sierpinski tetrahedron is a well-known fractal in three-dimensional space. When A is a regular tetrahedron with the vertices c_1, c_2, c_3, c_4, it is defined as the fixed point of the iteration function system (IFS) $\{f_1, f_2, f_3, f_4\}$ with f_i the dilation with the ratio $1/2$ and the center c_i ($i = 1, 2, 3, 4$). It is self-similar



(A) (B) Edge view (C) Vertex view

Fig. 1. Three views of the Sierpinski tetrahedron ([2]).

in that it is equal to the union of four half-sized copies of itself, and it is two dimensional with respects to fractal dimensions like the similarity dimension and the Hausdorff dimension. We refer the reader to [3] and [4] for the theory of fractals.

Fig. 1 shows some computer graphics images of the Sierpinski tetrahedron. As Fig. 1 (b) shows, it has a solid square image when projected from an edge, and there are three orthogonal directions in which the projection images become square. Here, we count opposite directions once. It is true both for the mathematically-defined pure Sierpinski tetrahedron and for its level n approximation model, which is obtained by applying the IFS n times starting with the tetrahedron A, and composed of 4^n regular tetrahedrons ($n \geq 0$).

While studying about generalizations of the Sierpinski tetrahedron, the author found a fractal in three-dimensional space which has *six* square projections [2]. This fractal and its finite approximation models are shown in Fig. 2. We start with a hexagonal bipyramid Fig. 2 (A, B, C). This dodecahedron is the intersection of a cube with its 60-degree rotation along a diagonal, and each of its face is an isosceles triangle whose height is 3/2 of the base. As (C) shows, it has square projections in six directions and it has square appearances when it is viewed from each of the 12 faces. We consider the IFS which is composed of nine dilations with the ratio 1/3 and the centers the 8 vertices and the center point of a hexagonal bipyramid. The pictures (D, E, F, G) and (H, I, J, K) are the 1st and 2nd level approximation models, respectively, and (L, M, N, O) are computer graphics of the hexagonal bipyramid fractal. As (G, K, O) shows, this fractal and its finite approximation models have six square projections.

In this paper, we consider the 2nd level approximation model. It consists of 81 hexagonal bipyramid pieces and, as (K) shows, it has 12 square appearances each of which is composed of $9 \times 9 = 81$ squares, which are divided into nine 3×3 blocks (P). This is nothing but the grid of the puzzle called SUDOKU or Number Place. The objective of this puzzle is to assign digits from 1 to 9 to the 81 squares so that each column, each row, and each of the nine 3x3 block contains all the nine digits. In SUDOKU puzzle, the digits do not have the meaning and we can use, for example, nine colors instead.

The goal of this paper is to find assignments of nine colors to the 81 pieces of the 2nd level approximation model of the hexagonal bipyramid fractal so that it has SUDOKU solution patterns in all the 12 square appearances. We also study SUDOKU coloring problem of level $2n$ approximation models $n \geq 1$.

Notation

When Δ is an alphabet and f is a function from Δ to Δ, we denote by $\mathrm{map}_n(f) : \Delta^n \to \Delta^n$ the component-wise application of f.

When $p \in \Delta^n$ and $q \in \Delta^m$ are sequences, we denote by $p \cdot q \in \Delta^{n+m}$ the concatenation of p and q, and by $q_{[i,j]}$ the subsequence of q from index i to j.

We use two alphabets $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $\Gamma = \{a, b, c, d, e, f, g, h, i\}$. Sequences of Σ are used for addresses, and sequences of Γ are used for colors.

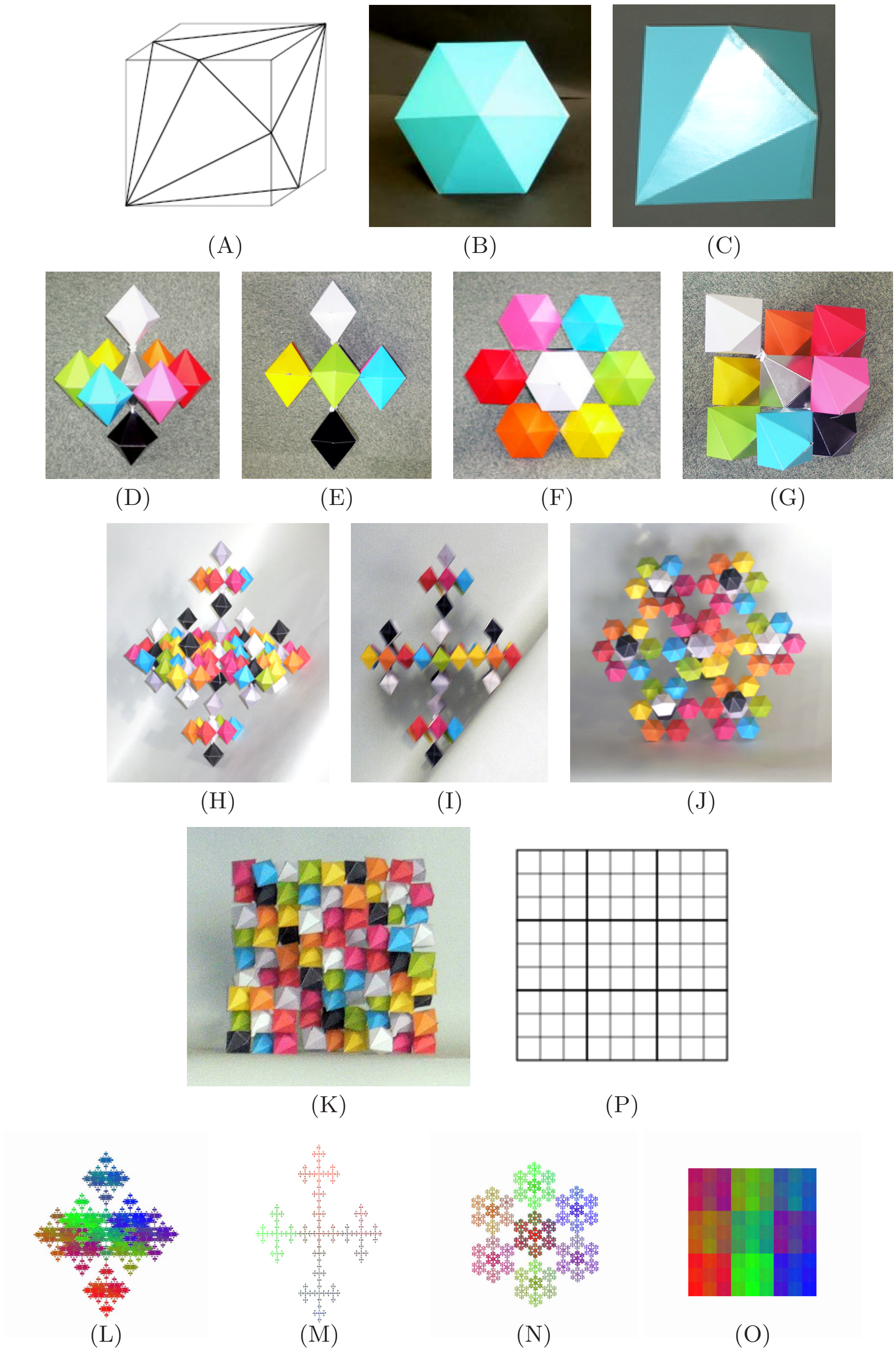


Fig. 2. The hexagonal bipyramid fractal and its approximations ([2]).

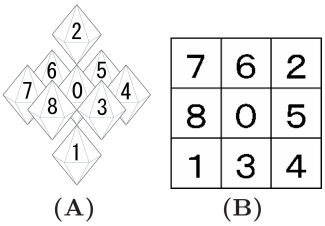


Fig. 3. Addressing of the level 1 object.

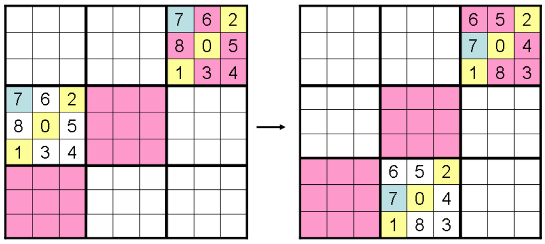


Fig. 4. Rotation of the level 2 object.

2 Mathematical Formulation

We can define SUDOKU coloring not only on the level 2 object but also on level $2n$ objects ($n \geq 1$). A level $2n$ object consists of 9^{2n} pieces and when it is viewed from each of its face, we have a $9^n \times 9^n$ grid of squares which consists of $3^n \times 3^n$-blocks arranged in a $3^n \times 3^n$ grid. Therefore, we have the coloring problem to assign 9^n colors so that each column, row, and $3^n \times 3^n$-block contain all the 9^n colors. We will first formalize this SUDOKU coloring problem symbolically as a problem of words.

We give addressing of the 1st level object with Σ as in Fig. 3 (A). We call the pieces 0, 1, 2 the axis pieces and 3, 4,..., 8 the ring pieces. The address of the level 2 object is given as pairs (a, b) for $a, b \in \Sigma$, where a specifies a block and b specifies a piece in the block. We call the blocks 0, 1, 2 the axis blocks and 3, 4,...,8 the ring blocks. Similarly, we give the addressing of the level $2n$ object with a tuple in Σ^{2n}. We sometimes fix a viewpoint so that the nine pieces of the 1st level object are arranged as Fig. 3 (B), and we specify a rotation of the object with a permutation on Σ^{2n}, instead of a changing of the viewpoint.

The symmetry group of a hexagonal bipyramid is the dihedral group D_6 of order 12. It is also true for its fractal and the finite approximation models. This group is composed of a rotation σ of 60 degree around the axis between block 1 and 2 and a rotation τ of 180 degree around the axis between block 4 and 7.

When σ is applied to the 1st level object, the axis pieces are fixed and the ring pieces are shifted to the next position. Therefore, it causes the permutation $\sigma_1 = (3\ 4\ 5\ 6\ 7\ 8)$ on Σ. A rotation on the 2nd level object is the combination of a revolution around the axis blocks and rotation inside each block, as Fig. 4 shows. In general, σ causes on the n-th level object a component-wise application of σ_1, which is $\sigma_n = \mathrm{map}_n(\sigma_1)$. We also consider the permutation corresponding to τ, which is defined for the 1st level object as $\tau_1 = (1\ 2)(6\ 8)(3\ 5)$ and for the level n object $\tau_n = \mathrm{map}_n(\tau_1)$. Thus, the symmetry group of the level n object is represented as $\langle \sigma_n, \tau_n \rangle$. We also define reflection $\upsilon_1 = (1\ 2)$ on the first level object and $\upsilon_n = \mathrm{map}_n(\upsilon_1)$ on the level n object.

Definition 1. *A coloring of the level* $2n$ *object is a function from* Σ^{2n} *to* Γ^n.

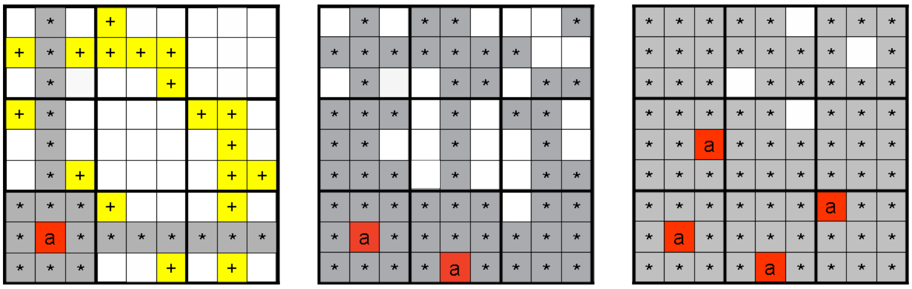


Fig. 5. Possible arrangement of one color 'a'.

We first define a SUDOKU coloring of a $9^n \times 9^n$-grid and then define a SUDOKU coloring of the level $2n$ object.

Definition 2. *A coloring $\gamma : \Sigma^{2n} \to \Gamma^n$ of the level $2n$ object is a one-face SUDOKU coloring when*
(1) The restriction of γ to $a_1 a_2 \dots a_n \Sigma^n$ is surjective for every $a_1 a_2 \dots a_n \in \Sigma^n$.
(2) For $A_1 = \{1,3,4\}, A_2 = \{8,0,5\}, A_3 = \{7,6,2\}$, the restriction of γ to $A_{d(1)} A_{d(2)} \dots A_{d(2n)}$ is surjective for every function $d : \{1,\dots,2n\} \to \{1,2,3\}$.
(3) For $B_1 = \{1,8,7\}, B_2 = \{3,0,6\}, B_3 = \{4,5,2\}$, the restriction of γ to $B_{d(1)} B_{d(2)} \dots B_{d(2n)}$ is surjective for every function $d : \{1,\dots,2n\} \to \{1,2,3\}$.

In condition (1), $a_1 \dots a_n$ specifies a $3^n \times 3^n$-block and this condition says that every $3^n \times 3^n$-block contains all the 9^n colors. In condition (2) and (3), a function $d : \{1,\dots,2n\} \to \{1,2,3\}$ specifies a row or a column and these conditions say that every row and every column contains all the 9^n colors.

Definition 3. *A coloring $\gamma : \Sigma^{2n} \to \Gamma^n$ is a SUDOKU coloring of the level $2n$ object if $\gamma \circ {\sigma_n}^k$ is a one-face SUDOKU coloring for $k = 0, 1, \dots, 5$.*

Note that $\gamma \circ \tau_n$ is always a one-face SUDOKU coloring when γ is, and we do not need to consider it in this definition.

Definition 4. *(1) Two colorings δ and η are change of colors when $\delta = p \circ \eta$ for a permutation p on Γ^n.*

(2) Two colorings δ and η are isomorphic when δ and $\eta \circ r$ are change of colors for r an element of the dihedral group D_6 generated by $\langle \sigma_{2n}, \tau_{2n} \rangle$.

(3) A coloring δ is a reflection of η when δ and $\eta \circ r$ are change of colors for $r \in \langle \sigma_{2n}, \tau_{2n}, \upsilon_{2n} \rangle$.

We identify colorings obtained by change of colors, and in this definition, we define isomorphism and reflection modulo change of colors.

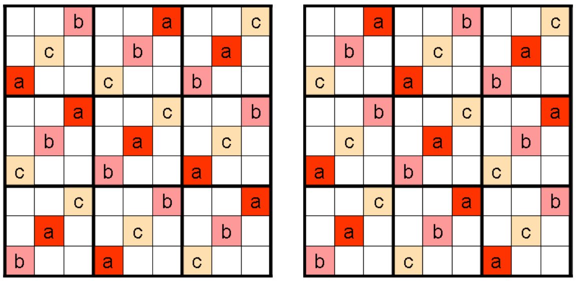


Fig. 6. Two solutions of the coloring of the axis pieces.

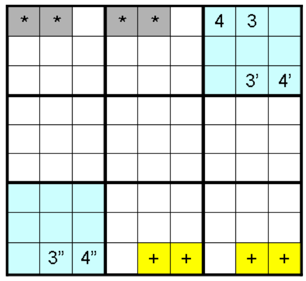


Fig. 7. Colorings of block 1 and 2.

3 SUDOKU Coloring of the 2nd level object

In this section, we determine all the SUDOKU colorings of the 2nd level object. Consider the assignment of color 'a' to the piece $(1, 0)$ as in Fig. 5 (A). In ordinary SUDOKU, we cannot assign the same color 'a' to pieces marked with $*$. In our SUDOKU coloring, we cannot assign it to pieces with $+$ either, because they come to places with $*$ by σ^k (k $= 1, \ldots, 5$). Note that $(1, 0)$ is on the axis of the axis block and therefore fixed by σ. As this example shows, the constraint we need to consider is very tight. Fig. 5 (B) shows the places 'a' can be assigned after it is assigned to $(1, 0)$ and $(3, 3)$. This figure shows that there is only one piece left in block 4 and 8, and when we have such an assignment, there is no piece left on block 5 (Fig. 5 (C)). Thus, the assignment of 'a' to $(3, 3)$ will cause a conflict. Similarly we cannot assign 'a' to $(3, 6)$. Therefore, $(3, 1)$ and $(3, 2)$ are the only pieces to which we can assign 'a', and they determine the left of the assignments of 'a' as in Fig. 6. Therefore, there are only two solutions for the assignment of 'a'. It is also the case for the colors assigned to $(0, 1)$ and $(0, 2)$, each of which has only two possible configurations, which are color 'b' and color 'c' in Fig. 6, respectively. As a consequence, we have only two colorings of the axis pieces, given in Fig. 6. Note that they switch by the application of σ, and also by the application of υ, modulus of change of colors.

It also shows that the SUDOKU coloring problem is the product of two independent coloring problems, one is the coloring of the 27 axis pieces with three colors 'a', 'b', and 'c', and the other one is the coloring of the 54 ring pieces with six colors 'd', 'e', 'f', 'g', 'h', and 'i'. We have shown that there are two isomorphic solutions to the former one.

Now, we study the coloring of the ring pieces. First, we study the two blocks 1 and 2. In Fig. 7, the four pieces marked with + (yellow) move to the places with * (gray) by σ^3, and pieces with the marks $3'$ and $4'$ move to 3 and 4 by σ^3. It means that the two colors assigned to $3''$ and $4''$ are different from the four colors assigned to the pieces marked with $+$, and thus equal to the two colors assigned to 3 and 4 after the application of σ^3, which are the colors of 3' and 4'. The same consideration applies to every pair of adjacent pieces on the rings

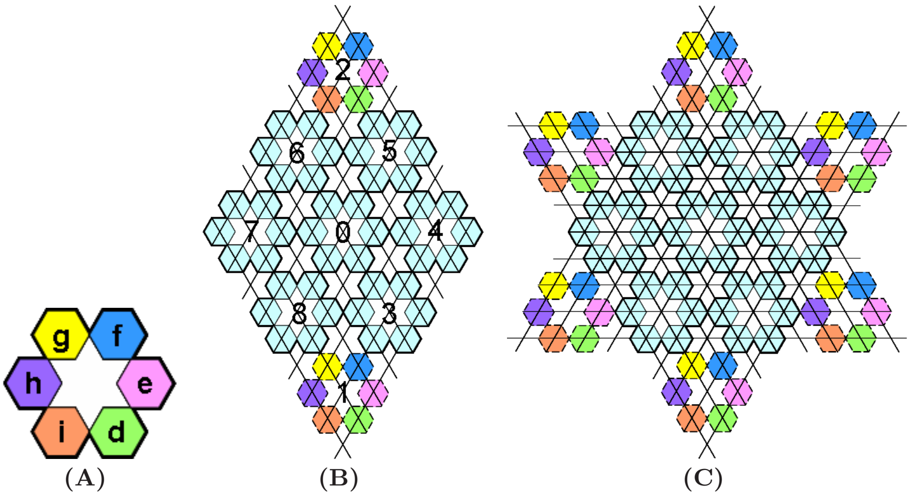


Fig. 8. Coloring constraint of the ring pieces of block 0, 3–8.

of blocks 1 and 2, and therefore, we can conclude that the color arrangements on the rings of these two blocks are the same. Since the coloring of the axis of these blocks are also the same in both colorings of the axis pieces, block 1 and 2 have the same coloring.

Next, we study the color arrangements of the ring pieces of the other seven blocks. In the followings, we fix the colors assigned to the pieces $(1, 3), \dots (1, 8)$ to be 'd', … 'i', respectively, as Fig. 8 (A) shows. In this figure, pieces $3, 4, 5, \dots$ are located from the lower-right corner anticlockwise. Since block 1 and 2 have the same coloring, the colors of the pieces $(2, 3)$ to $(2, 8)$ are also 'd', to 'i'. Each of the 18 lines of Fig. 8 (B) shows ring pieces which form the same row or column from the viewpoint of Fig. 3 (B). Fig. 8 (C) shows the rows and the columns of all the viewpoints in one figure, with the colorings of blocks 1 and 2 copied around the other blocks. In this figure, there are 27 lines. This figure shows the constraint we need to solve. That is, our goal is to assign 6 colors to the 42 pieces so that each of the 7 blocks and each of the 27 lines contain all the 6 colors.

Before studying the general case, we consider the case the center block (block 0) also has the same color assignment as block 1 and 2. In this case, according to the constraint, colors 'd', 'f', 'h' can appear on the pieces 3, 5, 7, and 'e', 'g', 'i' can appear on the pieces 4, 6, 8 of each block, respectively. Therefore, this coloring problem is the product of two coloring problems each of which is a coloring of 18 pieces with 3 colors. One can easily check that there are two solutions to each of them and we have 4 solutions as their composition. Since the two solutions alter by the application of σ, we have two solutions modulus of isomorphism (Fig. 9). Solution A has only one coloring pattern of a block, which is rotated by 120 degree to form three block patterns, which is assigned to blocks numbered with $(0, 1, 2), (3, 5, 7)$, and $(4, 6, 8)$, respectively. Solution B

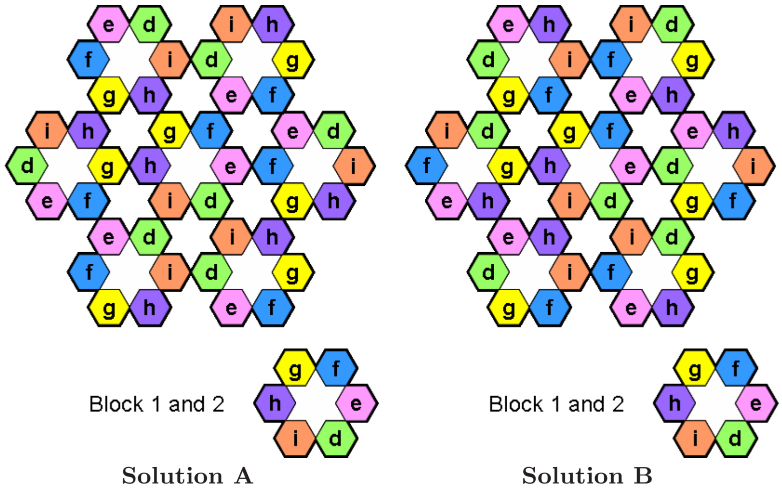


Fig. 9. Two ring colorings for the case the three axis blocks have the same coloring.

has three block coloring patterns each of which is assigned to three blocks as Solution A.

When block 0 is allowed to have different coloring from block 1 and 2, we have more solutions. Solution A in Fig. 9 has hexagonally arranged six pieces with two alternating colors which range over three blocks as Fig. 10 (A) shows. If we switch the two colors on these six pieces, the result also satisfy the constraint in Fig. 8 (C). There are three kinds of such hexagonal six pieces as Fig. 10 (A, B, C) shows. In these three figures, the vertices of each hexagon have two colors which can be switched. For each of these three, there are three places that the same kind of switching may occur. Since there is no overlapping, two or three switching may occur simultaneously as Fig. 10 (D, E) shows. Therefore, there are 3×3 non-isomorphic patterns of this kind. Patterns in Fig. 10 (A, B, C) can occur simultaneously only when they have the same center as in Fig. 10 (F), because there exist overlapping pieces for each of the other cases. Therefore, four kinds of combinations: (A, B), (B, C), (A, C), and (A, B, C) exist. Therefore, when we identify isomorphic colorings, we have $3 \times 3 + 4 = 13$ solutions except for solutions A and B.

Next, we count colorings which are isomorphic to one of them. Three solutions like (E) is mapped to itself by the application of σ^2 and τ, and therefore there are two non-isomorphic colorings, which switch by the application of σ. For the other ten patterns, the application of σ^k $(k = 0, 1.., 5)$ are all different and τ will map a coloring to one of them. Therefore, in all, there are $10 \times 6 + 3 \times 2 = 66$ solutions if we do not identify isomorphic ones.

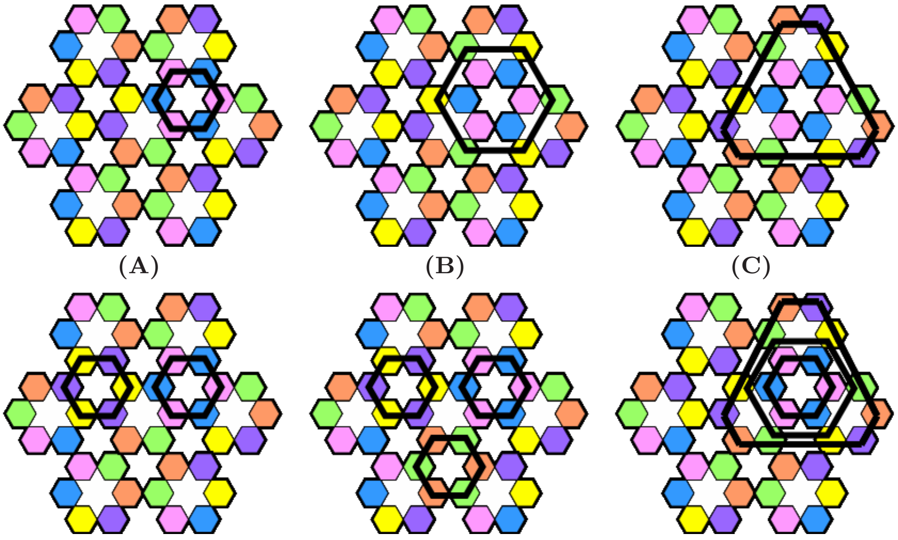


Fig. 10. Six pieces of Solution A, whose coloring can be switched.

As the result, we have $13 + 2 = 15$ solutions on the coloring of the rings modulus of isomorphism, and $66 + 4 = 70$ solutions if we do not identify isomorphic ones. Since there are two solutions on the coloring of the axis, we have 30 SUDOKU colorings of the level 2 model if we identify isomorphic ones, and 140 SUDOKU colorings if we do not identify isomorphic ones.

From the constraint of Fig. 8 (B), we can show that they are all the SUDOKU colorings of the level 2 object. From Fig. 8 (B), each ring piece of a ring block has three coloring possibilities as in Fig. 11. If we assume the color of (0, 3) (red piece in Fig. 11) to be 'f', then the possible colors of (3, 3) and (6, 3) are both 'h' and 'g', and therefore, 'h' is assigned to at least one of them. In the same way, 'h' must be assigned to one of (4, 3) and (7, 3), and one of (5, 3) and (8, 3). Therefore, 'h' is assigned to three pieces out of these six pieces, and one can show through some calculation that it causes a conflict. Therefore, it is not allowed to assign 'f' to (0, 3). In the same way, we cannot assign 'h' to (0, 3). Therefore, only 'd', 'e', 'g', 'i' are allowed to (0, 3). Among them, 'd' is the color assigned in Solution A and B. When 'e' is assigned to (0, 3), it is easily shown that the color of (0, 4) must be 'd'. Similarly, when 'g' is assigned to (0, 3), the color of (0, 6) must be 'd', and when 'i' is assigned to (0, 3), the color of (0, 8) must be 'd', In this way, we can show that the possible configurations of block 0 are only those obtained from that of Solution A through the switching listed in Fig. 10. Then, for each of the color assignment of block 0 obtained in this way,

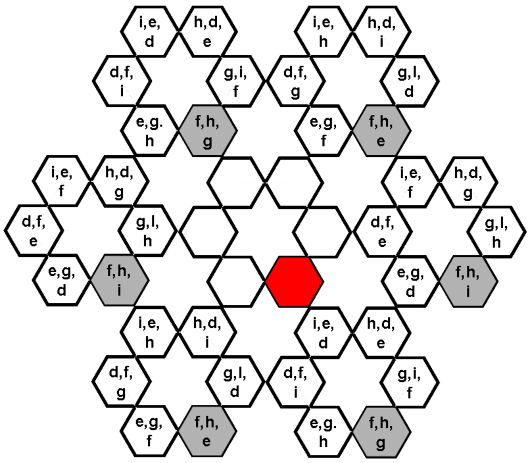


Fig. 11. The other 13 solutions on the ring.

we can calculate one or two color assignments of the other pieces, which are all among the solutions we have explained.

By the application of υ, the two axis colorings switch as we have noted, and the ring coloring is fixed because ring colorings on block 1 and 2 are the same. Therefore, the switching of the two axis-colorings causes reflection of the coloring.

Theorem 5. *The level 2 object has (1) 140 SUDOKU colorings if we identify change of colors, (2) 30 SUDOKU colorings if we identify isomorphic ones, (3) 15 SUDOKU colorings if we identify reflections.*

4 SUDOKU Coloring of the level $2n$ object

In this section, we show that SUDOKU colorings of the level $2n$ object exist for every $n \geq 1$.

Definition 6. *Let* $\gamma : \Sigma^{2n} \to \Gamma^n$ *and* $\delta : \Sigma^{2m} \to \Gamma^m$ *be colorings of the level* $2n$ *and the level* $2m$ *object, respectively. We define a coloring* $\mathrm{comp}(\gamma, \delta) : \Sigma^{2(n+m)} \to \Gamma^{n+m}$ *of the level* $2(n+m)$ *object as follows.*

$$\mathrm{comp}(\gamma, \delta)(p) = \gamma(p_{[m+1,m+2n]}) \cdot \delta(p_{[1,m]} \cdot p_{[m+2n+1,2m+2n]}) \, .$$

Proposition 7. *Suppose that* γ *and* δ *are SUDOKU colorings of the level* $2n$ *and the level* $2m$ *object, respectively. Then,* $\mathrm{comp}(\gamma, \delta,)$ *is a SUDOKU coloring of the level* $2(n+m)$ *object.*

Proof. We need to show that $\mathrm{comp}(\gamma,\delta)\circ{\sigma_{2(n+m)}}^k$ is a one-face SUDOKU coloring of level $2(n+m)$ for $k=0,1,\ldots,5$. However, since $\sigma_{2(n+m)}$ is the application of σ_1 to each component, we have $\sigma_{a+b}(p\cdot q)=\sigma_a(p)\cdot\sigma_b(q)$ for $p\in\Sigma^a$ and $q\in\Sigma^b$. Therefore,

$$\begin{aligned}
&\mathrm{comp}(\gamma,\delta)(\sigma_{2(n+m)}(p))\\
&\qquad=\mathrm{comp}(\gamma,\delta)(\sigma_m(p_{[1,m]})\cdot\sigma_{2n}(p_{[m+1,m+2n]})\cdot\sigma_m(p_{[m+2n+1,2m+2m]}))\\
&\qquad=\gamma(\sigma_{2n}(p_{[m+1,m+2n]}))\cdot\delta(\sigma_m(p_{[1,m]})\cdot\sigma_m(p_{[m+2n+1,2m+2n]}))\\
&\qquad=\gamma(\sigma_{2n}(p_{[m+1,m+2n]}))\cdot\delta(\sigma_{2m}(p_{[1,m]}\cdot p_{[m+2n+1,2m+2n]}))\\
&\qquad=\mathrm{comp}(\gamma\circ\sigma_{2n},\delta\circ\sigma_{2m})(p).
\end{aligned}$$

From our assumption, $\gamma\circ\sigma_{2n}$ and $\delta\circ\sigma_{2m}$ are one-face SUDOKU colorings of the level $2n$ and the level $2m$ object, respectively. Therefore, we only need to show that when γ and δ are one-face SUDOKU colorings of the level $2n$ and level $2m$ objects, respectively, then $\mathrm{comp}(\gamma,\delta)$ is a one-face SUDOKU coloring of the level $2(n+m)$ object.

Conditions (2) and (3) of Definition 2 hold because they are independent of the permutation of the coordinates. We show that condition (1) holds. Suppose that $p_1\in\Sigma^n$, $p_2\in\Sigma^m$, $r_1\in\Gamma^n$, and $r_2\in\Gamma^m$. We need to show that $r_1\cdot r_2=\mathrm{comp}(\gamma,\delta)(p_1\cdot p_2\cdot q)$ for some $q\in\Sigma^{n+m}$. Since γ and δ satisfy condition (1), there exists q_1 and q_2 such that $\gamma(p_1\cdot q_1)=r_1$ and $\gamma(p_2\cdot q_2)=r_2$ hold. Then, $\mathrm{comp}(\gamma,\delta)(p_2\cdot p_1\cdot q_1\cdot q_2)=\gamma(p_1\cdot p_2)\cdot\delta(q_1\cdot q_2)=r_1\cdot r_2$.

We proved in Section 3 that there are 30 SUDOKU colorings of level 2 object. Therefore, by induction we have the following.

Theorem 8. *There exist SUDOKU colorings of level $2n$ object for every $n\geq 1$.*

Usually, there are two ways of constructing level $n+1$ approximation models of a fractal from level n approximation models. One is to fix the size of the fundamental piece and construct a larger model by combining copies of the level n models. The other one is to fix the total size and replace the fundamental pieces of a level n model with level 1 approximation models. Correspondingly, there are two induction schemes to prove properties of approximation models of a fractal. In our proof of Proposition 7, we used both of them simultaneously. Let C_0 be a hexagonal bipyramid we start with. We consider the following construction of the level $2n+2$ approximation model C_{n+1} from the level $2n$ approximation model C_n. We replace each fundamental piece of C_n with a level 1 approximation model to form a level $2n+1$ approximation model B_n, then make 8 copies of B_n and locate them on the vertices of B_n so that B_n becomes the block 0 of the level $2n+2$ object C_{n+1}. Then, the size of the $3^n\times 3^n$-SUDOKU "blocks" of C_n is equal to the size of C_0 for every n. Our proof for the case $m=1$ constructs the SUDOKU coloring of C_{n+1} from that of C_n and a SUDOKU coloring of the 1st level object.

Corresponding to this construction, it is more natural to shift the index of the address space from $[0,2n]$ to $[-n+1,n]=\{-(n-1),-(n-2),\ldots,0,1,2,\ldots,n\}$. Then, the address grows in both directions when the level of the model increases, and the 9^n SUDOKU "blocks" are addressed with the index $[-(n-1),0]$ and inside each block, each piece is addressed with the index $[1,n]$.

Fig. 12. A SUDOKU Sculpture.

5 Conclusion

We showed that there are 140 SUDOKU colorings of the level 2 approximation of the hexagonal bipyramid fractal if we identify change of colors, and 30 if we identify isomorphic ones. This result is also verified by a computer program. Note that the ordinary SUDOKU has $18,383,222,420,692,992$ solutions if we identify change of colors [1]. Our SUDOKU problem has a strict constraint compared with the ordinal SUDOKU and therefore it is natural that we do not have many solutions. It is interesting to note that, as we have seen, all the solutions are symmetric to some extent and are logically constructed. It is in contrast to the lot of random-looking solutions of the ordinal SUDOKU.

Among the 30 SUDOKU coloring solutions, Solution A is most beautiful in that the order of the colors of the rings of the nine blocks are the same. The paper model in Figure 2 (H, I, J, K) has this coloring. This object is displayed at the Kyoto University Museum, with an acrylic resin frame object with 12 square colored faces. The 12 square faces are located so that this object looks square with SUDOKU pattern when it is viewed through these faces.

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