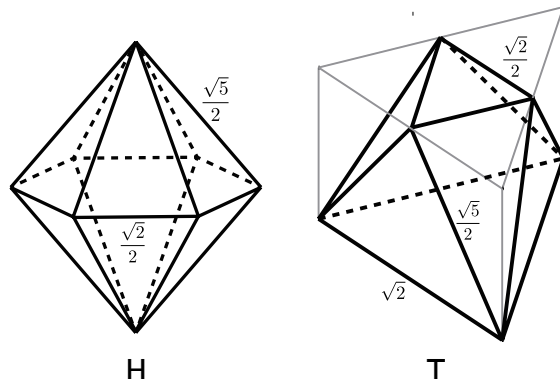


Imaginary Cubes H and T

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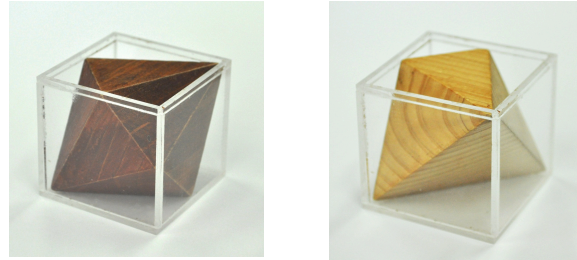
Imaginary Cubes are three dimensional objects which have square silhouette projections in three orthogonal ways just as a cube has. There are 16 equivalence classes of minimal convex imaginary cubes, among whose representatives are the following two objects as well as a regular tetrahedron and a cuboctahedron.



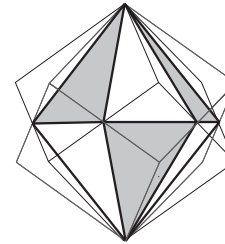
H (Hexagonal Bipyramid Imaginary Cube) is composed of two copies of a regular hexagonal pyramid whose side faces are isosceles triangles with the height $3/2$ of the base.

T (Triangular Antiprismoid Imaginary Cube) is obtained by truncating the three vertices of one base of a regular triangular prism whose height is $\sqrt{6}/4$ of an edge of a base. H and T are imaginary cubes with a lot of remarkable properties.

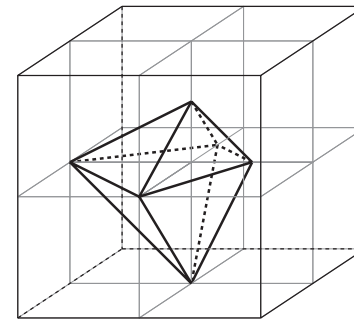
First, as imaginary cubes, they can be put into cubic boxes. If they are scaled as in the figure, they are put into unit cubes, with the vertices on cube-vertices and middle points of cube-edges.



H is the intersection of two cubes, and it is an imaginary cube of two cubes. Therefore, it has square projections not in 3 but in 6 ways.



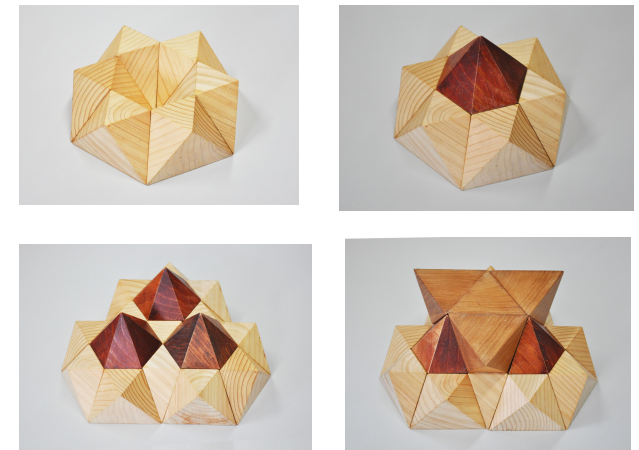
T has the property that the three diagonals intersect at one point and are orthogonal to each other. Thus, the six vertices are on the three axes and the distances from the origin are 1 on the negative side and $1/2$ on the positive side if they are scaled as in the figure.



Imaginary Cube Puzzle $3H=6T^*$ is a puzzle to put three copies of H and six copies of T (nine in all) into a double-sized cubic box. It is based on these properties of H and T.



A Tiling of the 3D Euclidean Space is formed by H and T. It is the Voronoi tessellation of the join of two cubic lattices which are 60-degree rotations of each other.



- References: (Available from <http://www.i.h.kyoto-u.ac.jp/~tsuiki/icube-e.html>)
1. Hideki Tsuiki. *Does It Look Square*. In Proceedings Bridges 2007, pp. 277-287.
 2. Hideki Tsuiki. *Imaginary Cubes*. In Proceedings Bridges 2010, pp.159-166.
 3. Hideki Tsuiki. *Imaginary Cubes and Their Puzzles*. (Draft, submitted for publication.)

(*) Will soon be available from Kyoto Univ. Museum Shop Musep.